ON THE PRESSURE UNDER AN INFINITE BEAM RESTING ON AN ELASTIC HALF-SPACE

(O DAVLENII POD BESKONECHNOI BALKOI LEZHASHCHEI NA UPRUGOM POLUPROSTRANSTVE)

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V. L. RVACHEV
(Berdiansk)

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The equation of the deformed axis of an infinite beam resting on an elastic half-space has been obtained in [1]. The present article presents formulas and tables by means of which the reaction of an elastic half-space can be determined at any point of the region of its contact with an infinite beam to which a concentrated force is applied. In conjunction with this a disagreement is found between the formulas here derived and those of Gorbunov-Posadov [2], according to whom the reaction pressure can be represented in the form

$$p(x, y) = \frac{p(y)}{\pi \sqrt{a^2 - x^2}} \tag{1}$$

where a is half the width of the beam. A comparison of numerical results shows, however, that this discrepancy is unimportant from the point of view of the practical application of the formulas. In the case where the beam is subjected to a concentrated force P, the deflection $w(y)$ of the axis of the beam is given by the formula

$$w(y) = \frac{Pa^3}{\pi EI} \int_0^\infty \frac{R(t)\cos(yt/a)\,dt}{t[\alpha + t^3 R(t)]} \left(\alpha = \frac{a^4 \pi E_0}{EJ(1 - \nu^2_0)}\right) \tag{2}$$

where EI is the rigidity of the beam, E_0 and ν_0 are constants characterizing the elastic properties of the foundation, $R(t)$ is some known function, and α is a dimensionless constant.

In order to find the reaction of the foundation, it is necessary to solve the contact problem for the pressure under a punch which has the plan-form of an infinite strip: $|x| \leqslant a$, $-\infty < y < +\infty$, with the condition that the surface of its foundation has the equation $z = w(y)$. (As usual, the beam is assumed to bend only in the longitudinal direction).

The writer's article [3] shows that the settling $w(x, y, 0) = b(\lambda)$

$\cos \lambda y$, $(|x| \leqslant a)$ corresponds to the pressure $p(x, y) = \phi(\lambda, x) \cos \lambda y$, $(|x| < 0)$, where

$$\varphi(\lambda, x) = \frac{E_0 b(\lambda)}{(1-\nu_0^2)\sqrt{a^2-x^2}} \sum_{k=0}^{\infty}{}' \delta_{2k}(a\lambda) \cos 2k \arccos \frac{x}{a} \tag{3}$$

In the last formula $\delta_{2k}(a\lambda)$ are some functions depending only on $a\lambda$; namely

$$\delta_{2k}(a\lambda) = (-1)^k \sum_{\nu=0}^{\infty}{}' \frac{A_0^{(2\nu)} A_{2k}^{(2\nu)} \operatorname{Fek}_{2\nu}'(0, -\frac{1}{4}a^2\lambda^2)}{\operatorname{Fek}_{2\nu}(0, -\frac{1}{4}a^2\lambda^2)} \tag{4}$$

where $\operatorname{Fek}_m(x, -q)$ are known Mathieu functions, $A_{2i}^{(2\nu)}$ are Fourier coefficients of the Mathieu functions $\operatorname{Ce}_{2\nu}(x, q)$, computed at $q = 1/4a^2\lambda^2$.

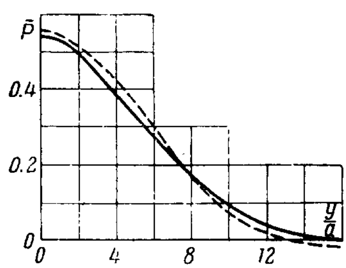
Because of formula (3), the settling (2) will correspond to the pressure

$$p(x, y) = \frac{PE_0 a^3}{\pi EI(1-\nu_0^2)\sqrt{a^2-x^2}} \sum_{k=0}^{\infty}{}' \gamma_{2k}(y) \cos 2k \arccos \frac{x}{a} \tag{5}$$

where

$$\gamma_{2k}(y) = \int_0^{\infty} \frac{R(t)\,\delta_{2k}(t)\cos(yt/a)\,dt}{t[\alpha + t^3 R(t)]} = \int_0^{\infty} \frac{\delta_{2k}(t)\cos(yt/a)\,dt}{\delta_0(t)[\alpha + t^3 R(t)]} \tag{6}$$

Here the equality $R(t) = t/\delta_0(t)$, which was presented in reference [1], has been used.



It is found that the terms of the series in formula (5) decrease so rapidly that in practice it is sufficient to confine oneself to the first two terms. Let

$$p(x, y) = \frac{P}{a^2\pi}\left(\frac{\alpha}{\pi}\right)^{1/3} p_0(x, y) \tag{7}$$

Then $p_0(x, y)$ will be a dimensionless quantity, depending on the dimensionless parameter α as well as on x and y. The figure shows a graph of $p_0(0, y)$ for

$$\beta = \sqrt[3]{\alpha/\pi} = 0.15,$$

i.e. when a = 0.0106. The dotted curve was constructed by using the Gorbunov-Posadov formulas and tables.

TABLE

t	$R(t)$	δ_2/δ_0	t	$R(t)$	δ_2/δ_0	t	$R(t)$	δ_2/δ_0
0	0	0	0.6	1.733	−0.0888	1.4	2.316	−0.215
0.1	0.624	−0.00434	0,7	1.846	−0.106	1.6	2.395	−0.240
0.2	0.977	−0.0163	0.8	1.944	−0.124	1.8	2.459	−0.263
0.3	1.235	−0.0333	0.9	2.027	−0.141	2.0	2.514	−0.284
0.4	1.436	−0.0523	1.0	2.100	−0.157	3,0	2.680	−0.365
0.5	1.600	−0.0710	1.2	2.213	−0.187	4.0	2.793	−0.419

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